

MEASURING Ω/b WITH WEAK LENSINGN. BENÍTEZ & J. L. SANZ^{1,2}

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ABSTRACT

A correlation between the surface density of foreground galaxies tracing the Large Scale Structure and the position of distant galaxies and quasars is expected from the magnification bias effect (Canizares 1981). We show that this foreground–background correlation function w_{fb} can be used as a straightforward and almost model-free method to measure the cosmological ratio Ω/b . For samples with appropriate redshift distributions, $w_{fb} \propto \Omega <\delta g>$, where δ and g are respectively the foreground dark matter and galaxy surface density fluctuations. Therefore, $\Omega/b \propto w_{fb}/w$, where $w \equiv <gg>$ is the foreground galaxy angular two-point correlation function, b is the biasing factor, and the proportionality factor is independent of the dark matter power spectrum. Simple estimations show that the application of this method to the galaxy and quasar samples generated by the upcoming Sloan Sky Digital Survey will achieve a highly accurate and well resolved measurement of the ratio Ω/b .

Subject headings: Cosmology; gravitational lensing; biasing factor

1. INTRODUCTION

Weak lensing promises to be one of the most effective ways to study the properties of Large Scale Structure (LSS) in the next years (Kaiser & Squires 1993; Kaiser 1998; Van Waerbeke, Bernardeau & Mellier 1999; Bartelmann & Schneider 1999). Most of the weak lensing methods proposed so far rest heavily on the analysis of background galaxy distortions (Kaiser 1992; Bernardeau et al. 1997, Schneider et al. 1997), as they potentially contain finer detail about the LSS than the fluctuations of the background number counts, which are affected by intrinsic clustering. However, detecting and analyzing the typical shear expected from the LSS offers technical problems difficult to underestimate (Kaiser, Squires & Broadhurst 1995; Bonnet & Mellier 1995; Van Waerbeke et al. 1997; Kaiser 1999; Kuijken 1999; although see Schneider et al 1998). In addition, a number of surveys will be available in the near future, as the Sloan digital Sky survey (Gunn & Weinberg 1995), NOAO (Januzzi et al. 1999) etc. with vast amounts of data which may not be optimal for shear analysis due to the imaging pixel size or typical seeing. It is therefore necessary to develop methods which are able to tap the wealth of cosmological information contained in such surveys without relying exclusively on image distortion analysis.

Such an approach is provided by the background-foreground correlation function w_{fb} (Bartelmann & Schneider 1991; Bartelmann & Schneider 1993). The value of this statistic has been calculated by several authors using linear and nonlinear evolution models for the power spectrum evolution (Bartelmann 1995; Villumsen 1996; Sanz, Martínez-González & Benítez 1997; Dolag & Bartelmann 1997; Moessner & Jain 1998; Moessner, Jain & Villumsen 1998). The two most obvious cases in which it is possible to measure w_{fb} are galaxy–galaxy correlations and quasar–galaxy correlations. The detection of the latter has a long and controversial history (for a discussion see Schneider, Ehlers & Falco 1992, Benítez 1997), but it seems to be already well established. However, due to the scarcity of complete, well defined quasar catalogs not affected by observational biases, the results have low signal-to-noise and are difficult to interpret (Benítez, Sanz & Martínez-González 1999). The value of the expected amplitude for the low- z galaxy–high- z galaxy cross-correlation is rather small, and hard to measure within typical single CCD fields (Villumsen, Freudling & Da Costa 1997). Only with the advent of deep, multicolor galaxy samples and reliable photometric redshift techniques it has been possible to detect this effect (Herranz et al. 1999).

To interpret the measurements of w_{fb} using the calculations mentioned above, it is necessary to assume a certain shape for the power spectrum. Unfortunately it is still far from clear whether the most popular

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ansatz, that of Peacock and Dodds (1996)—or any other for that matter—provides an accurate fit to the LSS distribution (see e.g. Jenkins et al. 1998). It is thus desirable to develop methods whose application is not hindered by this uncertainty. An example is the statistic R (Van Waerbeke, 1998), which combines shear and number counts information, and can be used to measure the scale dependence of the bias. The value of R is almost independent of the shape of the power spectrum if the foreground galaxy distribution has a narrow redshift range. Here we show that something similar can be achieved with w_{fb} , with the additional advantage of being able to do without the shear information in those cases where the latter is difficult to obtain.

It follows from the magnification bias effect (Canizares 1981, Narayan 1989, Broadhurst, Taylor & Peacock 1995) that the surface number density of background population n_b is changed by the magnification μ , associated with a foreground galaxy population with number density n as $g_b = (\alpha - 1)\delta\mu$, where g_b is the perturbation in the galaxy surface density n_b ($g_b = n_b / \langle n_b \rangle - 1$), α is the logarithmic slope of the number counts and in the weak lensing regime $\mu \approx 1 + \delta\mu$, $\delta\mu \ll 1$. Since $\mu \approx 1 + 2\kappa$ and κ , the convergence, is proportional to the projected matter surface density Σ , it follows that $g_b \propto \delta\kappa \propto \delta\Sigma \propto \Omega\delta$, where δ is the dark matter surface density perturbation. Therefore $w_{fb} \equiv \langle gg_b \rangle \propto \Omega \langle g\delta \rangle$ and assuming linear and deterministic bias $w_{fb} \propto \Omega b^{-1}w$ where w is the two-point galaxy correlation function for the background populations, and the biasing factor b is defined by $w = b^2 w_{\delta\delta}$. This result provides a straightforward, virtually model independent method to estimate the ratio Ω/b . Williams & Irwin 1998 arrived to a similar expression using a phenomenological approach. The redshift distortion method provides the quantity $\beta \propto \Omega^{0.6}/b$ (Dekel 1994), so combining both one can estimate Ω and b separately.

The outline of the paper is the following. In Sec. 2 we show rigorously that under reasonable assumptions $w_{fb} \propto \Omega b^{-1}w$. Sec. 3 explores the application of this method to future and ongoing surveys and Sec. 4 summarizes our main results and conclusions.

2. FOREGROUND-BACKGROUND CORRELATIONS

Let us consider two populations of sources: a background one (e.g. quasars or galaxies) and a foreground one (e.g. galaxies), placed at different distances λ with p.d.f.'s $R_b(\lambda)$ and $R(\lambda)$, respectively. In Sanz et al. (1997), the background-foreground correlation w_{fb} was calculated as a functional of the power spectrum $P(\lambda, k)$ of the matter fluctuations at any time (λ is the comoving distance from the observer to an object at redshift z , $\lambda = \frac{(1+\Omega z)^{1/2}-1}{(1+\Omega z)^{1/2}+1+\Omega}$ and $\Omega < 1, \Lambda = 0$):

$$w_{fb}(\theta) = (\alpha_b - 1)C_{\mu\delta}(\theta), \quad (1)$$

$$C_{\mu\delta}(\theta) = 12\Omega \int_0^1 d\lambda b(\lambda, s\theta) T_b(\lambda) R(\lambda) \frac{\lambda^2}{(1-\lambda)^2} \tau(\lambda, s\theta), \quad (2)$$

$$\tau(\lambda, s\theta) \equiv \frac{1}{2\pi} \int_0^\infty dk k P(\lambda, k) J_0(ks\theta), \quad s \equiv \frac{\lambda}{1 - (1-\Omega)\lambda^2}, \quad (3)$$

where $b(\lambda, s\theta)$ is the bias factor for the foreground galaxies (assumed to be linear, non-stochastic but possibly redshift and scale dependent), α_b is the slope of the background source number counts and $C_{\mu\delta}(\theta) \equiv 2 \langle \mu(\vec{\phi})\delta(\vec{\phi} + \vec{\theta}) \rangle$ is the correlation between the magnification and the mass density fluctuation. $T_b(\lambda)$ is the lensing window function given by equation (7) in Sanz et al. (1997):

$$T_b(\lambda) \equiv \frac{1}{\lambda} \int_\lambda^1 \frac{du}{u} R_b(u) (u - \lambda) \frac{1 - (1-\Omega)u\lambda}{1 - (1-\Omega)\lambda^2}.$$

The angular two point correlation function for the foreground population w can be obtained as

$$w(\theta) = \int_0^1 d\lambda b^2(\lambda, s\theta) R^2(\lambda) [1 - (1-\Omega)\lambda^2] \tau(\lambda, s\theta), \quad (4)$$

where $w(\theta) \equiv \langle g(\vec{\phi})g(\vec{\phi} + \vec{\theta}) \rangle$ is the foreground angular galaxy-galaxy correlation function.

If we assume that the foreground and background galaxies are concentrated at the ‘effective’ distances λ_f and λ_b (a good approximation for realistic, nonoverlapping redshift distributions, Sanz et al. 1997), the previous formulas can be rewritten as

$$w_{fb}(\lambda_f, \lambda_b, \theta) \simeq (\alpha_b - 1) 12\Omega \frac{2b}{a_f^2 \Sigma_c} \tau(\lambda_f, s_f \theta) \quad (5)$$

$$w(\lambda_f, \Delta\lambda_f, \theta) \simeq b^2 \frac{1}{\Delta\lambda_f} [1 - (1 - \Omega)\lambda_f^2] \tau(\lambda_f, s_f \theta), \quad (6)$$

where $\Sigma_c(\lambda_b, \lambda_f) \equiv \frac{2D_b}{D_f D_{fb}}$ is the critical surface mass distance defined by the angular distance D , $a_f \equiv \frac{(1-\lambda_f)^2}{1-(1-\Omega)\lambda_f^2}$ is the scale factor, and $\Delta\lambda_f \simeq 1/R_f(\lambda_f)$ is the width of the p.d.f. $R_f(\lambda)$ (e.g. the FWHM for a Gaussian distribution). Dividing Eq. (5) by Eq. (6) and simplifying one obtains

$$\frac{w_{fb}}{w} \simeq Q \Delta z_f (\alpha_b - 1) \frac{\Omega}{b} \quad (7)$$

i.e. the two correlations are proportional. The proportionality factor Q has the form

$$Q \equiv \frac{6\lambda_f(1 - \lambda_f/\lambda_b)[1 - (1 - \Omega)\lambda_f\lambda_b](1 - \lambda_f)}{[1 - (1 - \Omega)\lambda_f][1 - (1 - \Omega)\lambda_f^2]^2} \quad (8)$$

Fig. 1 (top) shows the dependence of Q on z_b and z_f for the $\Omega = 0.3, \Lambda = 0$ case. For the $\Omega + \Lambda = 1, \Lambda > 0$ case, there is not an explicit form for the angular distances, and the corresponding expression for Q is (Fig. 1, bottom)

$$Q = \frac{12(1 + z_f)}{\Sigma_c(z_f, z_b) \sqrt{1 + \Omega z_f + \Lambda[(1 + z_f)^{-2} - 1]}} \quad (9)$$

Note in Fig. 1 that for $z_f < 0.2$ and $z_b > 1$ the value of Q in both open and flat geometries changes very slowly with z_b and almost linearly with z_f . To better understand this behavior from Eq. (8) let's assume that $z_f \ll 1 < z_b$. In that case, $\lambda_f \simeq s_f \simeq \frac{1}{2}z_f$, $a_f \simeq 1 - z_f$ and $Q \approx 3z_f$. Therefore, w_{fb} is almost independent of λ_b (Fig. 1) and roughly $w_{fb} \simeq 3z_f \Delta z_f (\alpha_b - 1) \Omega b^{-1} w$. Fig. 2 also shows that for the same redshift range, the amplitude of Q is also approximately independent of Ω , allowing an empirical, model-independent estimation of Ω/b .

3. PRACTICAL APPLICATION

The main quantity which determines the signal in the measurement of the angular correlation function is the excess (or defect) in the expected number of galaxy pairs. For a bin with surface A_{bin} at distance θ this number will be

$$\Delta N(\theta) \approx (\pm) N_f n_b A_{bin}(\theta) w_{fb}(\theta) \quad (10)$$

Therefore, the signal-to-noise of the detection will be roughly

$$\frac{S}{N} \sim \frac{\Delta N}{\sqrt{N}} = \sqrt{N_f} \sqrt{n_b A_{bin}} w_{fb}(\theta) \quad (11)$$

We have not included the scatter due to the clustering of foreground galaxies and background galaxies, but it is unlikely that this effect will increase the noise over Poisson more than a factor of a few for any realistic case. The quantity $n_b A_{bin}$ is the number of background galaxies per bin. If we assume radial concentric bins of width $\Delta\theta$ and $w_{fb} = A_{fb} \theta^{-\gamma}$, the signal to noise in each bin will be

$$\frac{S}{N} \sim A_{fb} \sqrt{2\pi N_f n_b \Delta\theta} \theta^{0.5-\gamma} \quad (12)$$

Since γ is typically $0.7 - 0.8$, the efficiency of the method will decrease very slowly with radius, allowing to map Ω/b up to very large scales.

The SDSS will obtain $\sim 10^6$ spectra for a population of $\langle z \rangle \sim 0.1$ galaxies (Gunn & Weinberg 1995), forming a splendid foreground sample. It may be assumed that its projected angular correlation function will be similar to that of the APM catalog, which has an amplitude $A = 0.44$ at $1'$ and a slope $\gamma = 0.668$ (Maddox et al. 1990). The SDSS will also obtain $S/N \sim 10$ $u'g'r'i'z'$ photometry for 10^8 galaxies in a 10^4 degree region, for which photometric redshifts will be estimated. The background sample can be formed by those galaxies with $z > 0.4$ and $\langle z \rangle \approx 0.7 - 0.8$, with $n_b \approx 1 - 1.5/\square'$. The value of Q cannot be considered constant for these two samples (see Fig. 1) because of the range of redshifts involved. Luckily, the existence of spectroscopic

redshifts for the foreground sample will allow to select subsamples with an extremely thin redshift distribution for which Q is approximately constant, each yielding independent estimates of Ω/b which can afterwards be combined together. A rough estimate of the expected result using Eq. (12) gives

$$\left(\frac{S}{N}\right)_{SDSS} \sim 20 \frac{\Omega}{b} \sqrt{\left(\frac{\Delta\theta}{5'}\right)} \theta^{-0.2} \quad (13)$$

(we have assumed $N_f = 10^6$, $n_b = 1.5$, $\Delta z_f = 0.05$, $Q = 0.25$, $\alpha_b - 1 = 0.5$) This shows that it will be possible to map Ω/b with an extremely good combination of resolution and accuracy using the SDSS. At large radii, the bias factor is expected to become constant (Coles 1993), which means that the bin size can be made as large as desired without being affected by the scale dependence of b and therefore substantially decrease the error in the estimation of Ω/b . It is obvious from the above numbers that the main source of errors will not be the shot noise, but contamination due to low redshift galaxies which may sneak into the high-redshift sample creating a spurious correlation. This contamination can be very effectively minimized by applying a Bayesian threshold (Benítez 1999). In addition, it will be possible to accurately quantify and correct this effect using a calibration sample with spectroscopic redshifts.

The SDSS also plans to obtain spectra for a sample of 10^5 red luminous galaxies with $< z > \sim 0.4$ and 10^5 QSOs. Due to the much lower density of the background sample, it will not be possible to attain such a good spatial resolution in the measurement of Ω/b , but these samples will allow to trace the evolution of biasing with redshift up to $z \sim 1$. However, experience shows that in this case the utmost attention will have to be paid to eliminating or accounting for the observational biases in the QSO detection and identification procedure, which can totally distort the estimation of w_{bf} (Ferrerias, Benítez & Martínez-González 1997, Benítez, Sanz & Martínez-González 1999).

4. CONCLUSIONS

A correlation (positive or negative) between the surface density of foreground galaxies tracing the Large Scale Structure and the position of background galaxies and quasars is expected from the magnification bias effect (Canizares 1981). We show that this foreground-background correlation function w_{fb} can be used as a straightforward and almost model-free method to measure the cosmological ratio Ω/b .

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Simple estimations show that the application of this method to the galaxy and quasar samples generated by the upcoming Sloan Sky Digital Survey will achieve a highly accurate and well resolved measurement of the ratio Ω/b .

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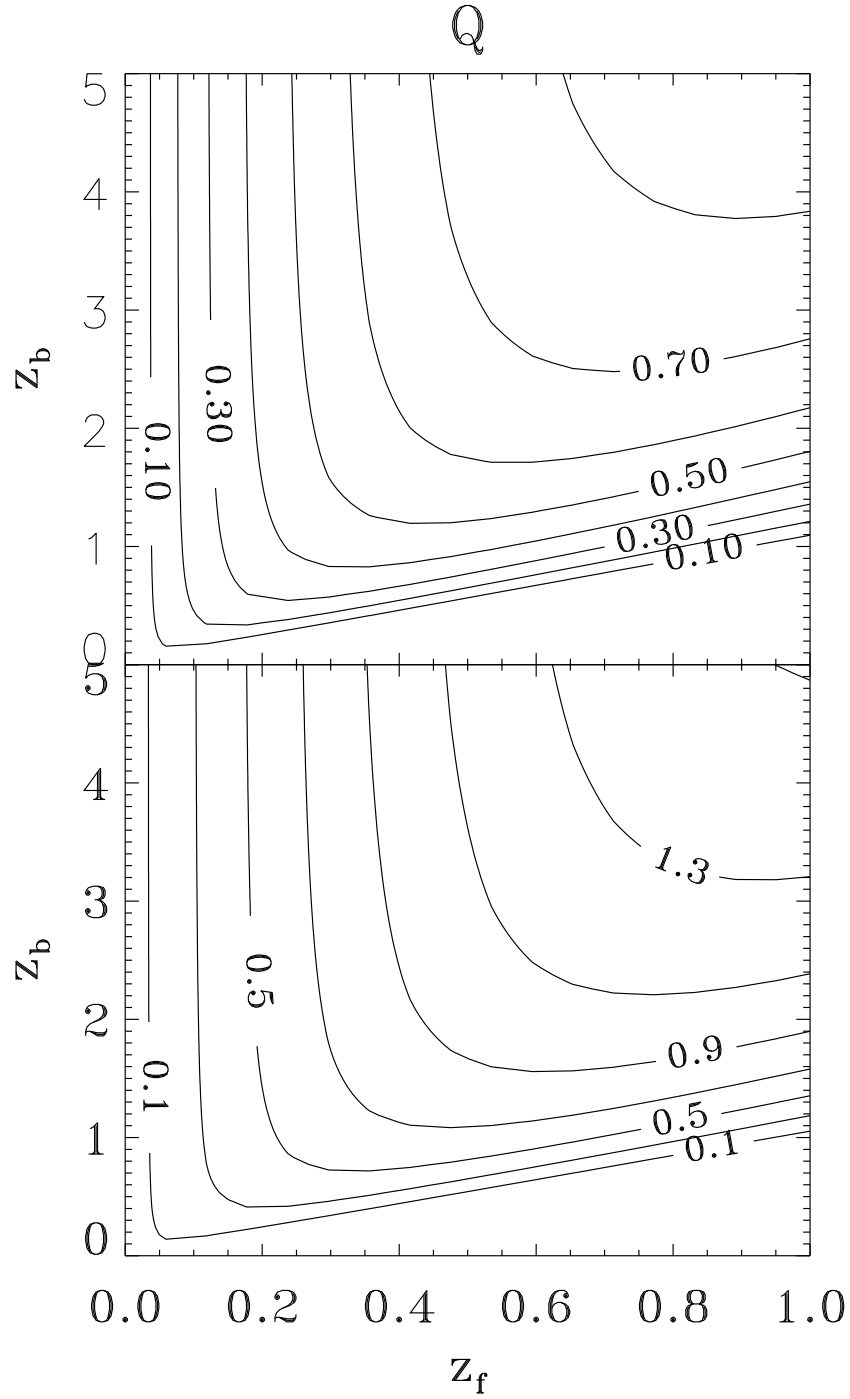


FIG. 1.— Dependence of the factor Q (Eqs. 8, 11) on the redshift of the foreground and background samples for open (top, $\Lambda = 0$) and flat (bottom, $\Omega + \Lambda = 1$) geometries ($\Omega = 0.3$ in both cases).

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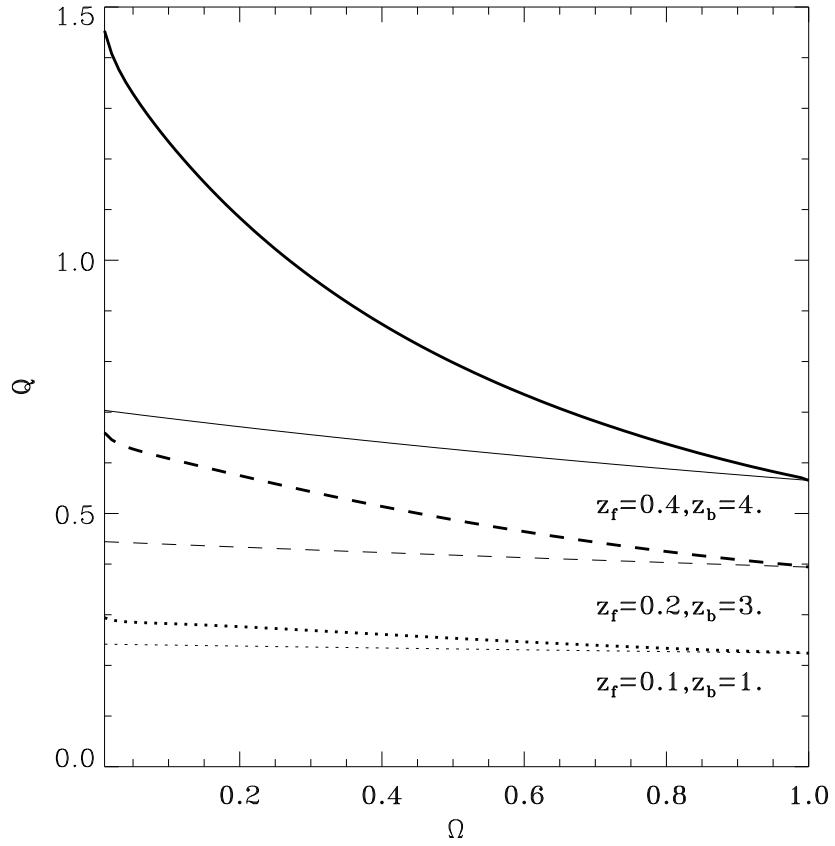


FIG. 2.— Dependence of the factor Q (Eqs. 8, 11) on Ω for different redshifts of the foreground and background populations z_f and z_b . The thick lines correspond to $\Omega + \Lambda = 1$ universes and the thin ones to $\Lambda = 0$.